Marking Scheme
Strictly Confidential
(For Internal and Restricted use only)
Senior Secondary School Examination 2026
MATHEMATICS (041) (PAPER CODE -65/3/3)

General Instructions: -

1	You are aware that evaluation is the most important process in the actual and correct assessment of the candidates. A small mistake in evaluation may lead to serious problems which may affect the future of the candidates, education system and teaching profession. To avoid mistakes, it is requested that before starting evaluation, you must read and understand the spot evaluation guidelines carefully.
2	"Evaluation policy is a confidential policy as it is related to the confidentiality of the examinations conducted, Evaluation done and several other aspects. Its leakage to the public in any manner could lead to derailment of the examination system and affect the life and future of millions of candidates. Sharing this policy/document to anyone, publishing in any magazine and printing in Newspaper/Website, etc. may invite action under various rules of the Board and BNS."
3	Evaluation is to be done as per instructions provided in the Marking Scheme. It should not be done according to one's own interpretation or any other consideration. The Marking Scheme should be strictly adhered to and religiously followed. However, while evaluating, answers which are based on latest information or knowledge and/or are innovative, they may be assessed for their correctness otherwise and due marks be awarded to them. In class-XII, while evaluating the competency-based questions, please try to understand the given answer and even if reply is not from a marking scheme but correct competency is enumerated by the candidate, due marks should be awarded.
4	The Marking Scheme carries only suggested value points for the answers. These are Guidelines only and do not constitute the complete answer. The students can have their own expression and if the expression is correct, the due marks should be awarded accordingly.
5	The Head-Examiner must go through the first five answer books evaluated by each evaluator on the first day, to ensure that evaluation has been carried out as per the instructions given in the Marking Scheme. If there is any variation, the same should be zero after deliberation and discussion. The remaining answer books meant for evaluation shall be given only after ensuring that there is no significant variation in the marking of individual evaluators.
6	Evaluators will mark (√) wherever answer is correct. For wrong answer CROSS 'X' be marked. Evaluators will not put right (✓) while evaluating which gives the impression that the answer is correct, and no marks are awarded. This is the most common mistake which evaluators are committing.
7	If a question has parts, please award marks on the right-hand side for each part. Marks awarded for different parts of the question should then be totaled up and written in the left-hand margin and encircled. This may be followed strictly.
8	If a question does not have any parts, marks must be awarded in the left-hand margin and encircled. This may also be followed strictly.
9	If a student has attempted an extra question, answer to the question deserving more marks should be retained and the other answer scored out with a note "Extra Question".
10	No marks to be deducted for the cumulative effect of an error. It should be penalized only once.

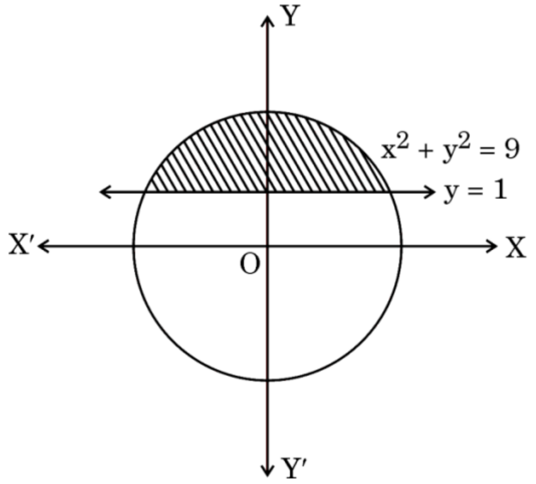
11	**A full scale of marks (example 0 to 80/70/60/50/40/30 marks as given in Question Paper) has to be used. Please do not hesitate to award full marks if the answer deserves it.**
12	**Every examiner must necessarily do evaluation work for full working hours, i.e., 8 hours every day and evaluate 20 answer books per day in main subjects and 25 answer books per day in other subjects (Details are given in Spot Guidelines). This is in view of the reduced syllabus and number of questions in question paper.**
13	**Ensure that you do not make the following common types of errors committed by the Examiner in the past: -** ● **Leaving answer or part thereof unassessed in an answer book.** ● **Giving more marks for an answer than assigned to it.** ● **Wrong totaling of marks awarded on an answer.** ● **Wrong transfer of marks from the inside pages of the answer book to the title page.** ● **Wrong question wise totaling on the title page.** ● **Wrong totaling of marks of the two columns on the title page.** ● **Wrong grand total.** ● **Marks in words and figures not tallying/not same.** ● **Wrong transfer of marks from the answer book to online award list.** ● **Answers marked as correct, but marks not awarded. (Ensure that the right tick mark is correctly and clearly indicated. It should merely be a line. Same is with the X for incorrect answer.)** ● **Half or a part of the answer marked correct and the rest as wrong, but no marks awarded.**
14	**While evaluating the answer books if the answer is found to be totally incorrect, it should be marked as cross (X) and awarded zero (0) Marks.**
15	**Any unassessed portion, non-carrying over of marks to the title page, or total error detected by the candidate shall damage the prestige of all the personnel engaged in the evaluation work as also of the Board. Hence, to uphold the prestige of all concerned, it is again reiterated that the instructions be followed meticulously and judiciously.**
16	**The Examiners should acquaint themselves with the guidelines given in the "Guidelines for Spot Evaluation" before starting the actual evaluation.**
17	**Every Examiner shall also ensure that all the answers are evaluated, marks carried over to the title page, correctly totaled and written in figures and words.**
18	**The candidates are entitled to obtain a photocopy of the Answer Book on request on payment of the prescribed processing fee. All Examiners/Additional Head Examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.**

MARKING SCHEME
MATHEMATICS (Subject Code–041)
(PAPER CODE: 65/3/3)

Q. No.	EXPECTED OUTCOMES/VALUE POINTS	STEPS	MARKS
	SECTION A This section comprises 20 multiple choice questions (MCQs) of 1 mark each.		
1.	If $(\vec{a}+\vec{b}).(\vec{a}-\vec{b}) = 198$ and $\lvert\vec{a}\rvert = 10\lvert\vec{b}\rvert$, then : (A) $\lvert\vec{a}\rvert = \sqrt{2}$ (B) $\lvert\vec{b}\rvert = \sqrt{2}$ (C) $\lvert\vec{b}\rvert = 10\sqrt{2}$ (D) $\lvert\vec{a}\rvert = \frac{10}{\sqrt{2}}$		
Sol. 1	(B) $\lvert\vec{b}\rvert = \sqrt{2}$	**I**	1
2.	If l_1, m_1, n_1 and l_2, m_2, n_2 are direction cosines of lines L_1 and L_2 respectively and θ is the acute angle between them, then : (A) $\cos\theta = l_1l_2 + m_1m_2 + n_1n_2$ (B) $\sin\theta = l_1l_2 + m_1m_2 + n_1n_2$ (C) $\tan\theta = \frac{l_1}{l_2} + \frac{m_1}{m_2} + \frac{n_1}{n_2}$ (D) $\cos\theta = \lvert l_1l_2 + m_1m_2 + n_1n_2 \rvert$		
Sol. 2	(D) $\cos\theta = \lvert l_1l_2 + m_1m_2 + n_1n_2\rvert$	**I**	1
3.	Direction ratios of lines l_1 and l_2 respectively are <1, 0, 0> and <0, –1, 0>. The direction ratios of the line perpendicular to both l_1 and l_2 are : (A) <1, 1, 0> (B) <0, 0, –1> (C) <1, 1, 1> (D) <1, 0, –1>		
Sol. 3	(B) $< 0, 0, -1 >$	**I**	1
4.	If E and F are two independent events such that $P(E) = \frac{3}{10}$, $P(E \cup F) = \frac{1}{2}$, then $P(E\mid F) - P(F\mid E)$ is equal to : (A) $\frac{2}{7}$ (B) $\frac{3}{35}$ (C) $\frac{1}{70}$ (D) $\frac{1}{7}$		
Sol. 4	(C) $\frac{1}{70}$	**I**	1
5.	The domain of $\cos^{-1}(4x+1)$ is : (A) $[-1, 1]$ (B) $[-3, 5]$ (C) $[-4, 4]$ (D) $[-\frac{1}{2}, 0]$		
Sol. 5	(D) $[-\frac{1}{2}, 0]$	**I**	1

6.	If $A^2 = 4A + 3I$ and $A^{-1} = xA + yI$, then the value of $(x + y)$ is : (A) -1 (B) 1 (C) $\frac{5}{3}$ (D) 7		
Sol. 6	(A) -1	**I**	1
7.	If A and B are skew-symmetric matrices of same order, then $AB' + BA'$ is a/an : (A) symmetric matrix (B) skew-symmetric matrix (C) null matrix (D) identity matrix		
Sol. 7	(A) symmetric matrix	**I**	1
8.	If a matrix X is such that $[2 \quad 1]\, X = [3 \quad 4 \quad 5]$, then the order of matrix X is : (A) 3×1 (B) 2×3 (C) 1×2 (D) 1×3		
Sol. 8	(B) 2×3	**I**	1
9.	If a square matrix A is such that $A^2 = A$ and $(I - A)^3 = xA + I$, then value of x must be : (A) 7 (B) 5 (C) -7 (D) -1		
Sol. 9	(D) -1	**I**	1
10.	If $A(\text{adj } A) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then the value of $\lvert 2A \rvert$ is : (A) 6 (B) 54 (C) 12 (D) 24		
Sol. 10	(D) 24	**I**	1
11.	The value of k for which the function $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ k(x+1), & x = 0 \end{cases}$ is a continuous function, is : (A) $\frac{1}{4}$ (B) 2 (C) $\frac{1}{2}$ (D) 0		
Sol. 11	(D) 0	**I**	1

12.	If $2\cos^{-1}x = y$, then $\frac{dy}{dx}$ is : (A) $-2\sin^{-1}x$ (B) $-\frac{1}{2}\sin\frac{y}{2}$ (C) $\frac{-1}{\sqrt{1-2x^2}}$ (D) $-2\operatorname{cosec}\frac{y}{2}$		
Sol. 12	(D) $-2\operatorname{cosec}\frac{y}{2}$	**I**	1
13.	The rate of change of volume of a sphere with respect to its diameter, when its radius is 5 cm, is : (A) $400\pi\ cm^3/cm$ (B) $100\pi\ cm^3/cm$ (C) $50\pi\ cm^3/cm$ (D) $25\pi\ cm^3/cm$		
Sol. 13	(C) $50\pi\ cm^3/cm$	**I**	1
14.	$\int \frac{\cos x}{\sqrt{\sin^2 x+1}}\,dx$ is equal to : (A) $\tan^{-1}(\sin x) + C$ (B) $\sin^{-1}(\sin x) + C$ (C) $\log\lvert \sin x + \sqrt{1+\sin^2 x}\ \rvert + C$ (D) $\log\lvert \sin x - \sqrt{\sin^2 x+1}\ \rvert + C$		
Sol. 14	(C) $\log\lvert \sin x + \sqrt{1+\sin^2 x}\rvert + C$	**I**	1
15.	$\int_{-1}^{1}(1-\lvert x\rvert)\,dx$ is equal to : (A) $2\int_{0}^{1}(1+x)\,dx$ (B) $2\int_{-1}^{0}(1+x)\,dx$ (C) 0 (D) $2\int_{-1}^{0}(1-x)\,dx$		
Sol. 15	(B) $2\int_{-1}^{0}(1+x)\,dx$	**I**	1

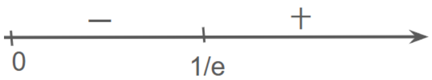
16.	The area of the shaded region of the circle given below is equal to :  (A) $\int_1^3 \sqrt{9 - y^2}\, dy$ (B) $2\int_1^3 \sqrt{9 - y^2}\, dy$ (C) $\int_0^3 \sqrt{9 - x^2}\, dx$ (D) $2\int_0^3 \sqrt{9 - x^2}\, dx$		
Sol. 16	(B) $2\int_1^3 \sqrt{9 - y^2}\, dy$	**I**	1
17.	$\frac{dy}{dx} = F(x, y)$ will be a homogeneous differential equation for which of the following functions ? (i) $F(x, y) = 3x + 2y$ (ii) $F(x, y) = \sin \frac{y}{x} + \log y - \log x$ (iii) $F(x, y) = e^{y/x} + 1$ (iv) $F(x, y) = \sqrt{x^2 + y^2} - y$ (A) (i) and (ii) (B) (i), (ii) and (iii) (C) (ii), (iii) and (iv) (D) (ii) and (iii)		
Sol. 17	(D) (ii) and (iii)	**I**	1
18.	For any two vectors $\vec{a}$ and $\vec{b}$, which of the following statements is always true ? (A) $\vec{a} \cdot \vec{b} \leq \|\vec{a}\|\,\|\vec{b}\|$ (B) $\|\vec{a} + \vec{b}\| \geq \|\vec{a}\| + \|\vec{b}\|$ (C) $\|\vec{a} - \vec{b}\| = \|\vec{a}\| - \|\vec{b}\|$ (D) $\|\vec{a} \times \vec{b}\| \geq \|\vec{a}\|\,\|\vec{b}\|$		
Sol. 18	(A) $\vec{a} \cdot \vec{b} \leq \|\vec{a}\|\,\|\vec{b}\|$	**I**	1
Questions number **19** and **20** are Assertion and Reason based questions. Two statements are given, one labelled as Assertion (A) and the other labelled as Reason (R). Select the correct answer from the codes (A), (B), (C) and (D) as given below.			

	(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A). (B) Both Assertion (A) and Reason (R) are true, but Reason (R) is **not** the correct explanation of the Assertion (A). (C) Assertion (A) is true, but Reason (R) is false. (D) Assertion (A) is false, but Reason (R) is true.		
19.	*Assertion (A) :* One of the particular solutions of the differential equation $\frac{dy}{dx} = e^{x+y}$ can be $e^x + e^{-y} = -2$. *Reason (R) :* $e^x + e^{-y} = C$ is the general solution of the differential equation $\frac{dy}{dx} = e^{x+y}$.		
Sol. 19	(D) Assertion (A) is false, but Reason (R) is true.	**I**	1
20.	*Assertion (A) :* The vectors $\vec{a}$ and $(-2\vec{a})$, where $\vec{a} \neq \vec{0}$ are collinear vectors. *Reason (R) :* $\vec{a} \cdot (-2\vec{a}) = 0$.		
Sol. 20	(C) Assertion (A) is true, but Reason (R) is false.	**I**	1

SECTION B

This section comprises 5 Very Short Answer (VSA) type questions of 2 marks each.

21.	(a) If the lines $\frac{x-3}{1} = \frac{1-y}{1} = \frac{z+2}{p}$ and $\frac{2-x}{3} = \frac{y+1}{5} = \frac{z+56}{2p}$ are perpendicular to each other, then find the value(s) of p. **OR** (b) Find the vector equation of a line passing through the origin and perpendicular to both the lines $\vec{r} = 2\hat{i} - \hat{j} + 2\hat{k} + \lambda(3\hat{i} + 4\hat{j} + 2\hat{k})$ and $\vec{r} = \mu(\hat{i} - \hat{j} + \hat{k})$.		
Sol. 21(a)	Direction Ratios of line $\frac{x-3}{1} = \frac{1-y}{1} = \frac{z+2}{p}$ are $< 1, -1, p >$	**I**	½
	Direction Ratios of line $\frac{2-x}{3} = \frac{y+1}{5} = \frac{z+56}{2p}$ are $< -3, 5, 2p >$	**II**	½
	$\because$ Given both lines are perpendicular to each other. $\therefore 1 \times (-3) + (-1) \times 5 + p \times 2p = 0$	**III**	½

(b)	$\Rightarrow p = \pm 2$ **OR** The vector parallel to required line is given by $\vec{b} = (3\hat{i} + 4\hat{j} + 2\hat{k}) \times (\hat{i} - \hat{j} + \hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 2 \\ 1 & -1 & 1 \end{vmatrix} = 6\hat{i} - \hat{j} - 7\hat{k}$ Equation of required line is given by $\vec{r} = \delta(6\hat{i} - \hat{j} - 7\hat{k})$	**IV** **I** **II**	½ $1\frac{1}{2}$ ½
22.	If $x = e^{\sin^{-1} t}$ and $y = e^{\cos^{-1} t}$, then find $\frac{dy}{dx}$ at $t = \frac{1}{\sqrt{2}}$.		
Sol. 22	$x = e^{\sin^{-1}t} \Rightarrow \frac{dx}{dt} = \frac{1}{\sqrt{1-t^2}} e^{\sin^{-1}t}$ $y = e^{\cos^{-1}t} \Rightarrow \frac{dy}{dt} = -\frac{1}{\sqrt{1-t^2}} e^{\cos^{-1}t}$ $\frac{dy}{dx} = -\frac{e^{\cos^{-1}t}}{e^{\sin^{-1}t}}$ $\left[\frac{dy}{dx}\right]_{t=\frac{1}{\sqrt{2}}} = -1$	**I** **II** **III** **IV**	½ ½ ½ ½
23.	Find the sub-interval of $(0, \infty)$ in which $f(x) = x \log x$ is increasing.		
Sol. 23	$f(x) = x \log x \Rightarrow f'(x) = 1 + \log x$ Put $1 + \log x = 0 \Rightarrow x = \frac{1}{e}$  $\because f'(x) > 0 \ \forall x \in \left(\frac{1}{e}, \infty\right) \Rightarrow f(x)$ is increasing in $\left(\frac{1}{e}, \infty\right)$. **Note:** Interval $[1/e, \infty)$ can be considered.	**I** **II** **III**	1 ½ ½
24.	(a) Find the value of $\sin[\cot^{-1}\sqrt{2}\,(\cos(\tan^{-1}1))]$. **OR** (b) A relation R on A = {1, 2, 3} is defined as R = {(1, 1) (3, 3), (1, 2)}. Is R a symmetric relation ? Justify. Write the smallest relation set R_1 such that $R \cup R_1$ becomes an equivalence relation on the set {1, 2, 3}.		
Sol. 24(a)	$\sin[\cot^{-1}\sqrt{2}(\cos(\tan^{-1}1)] = \sin\left[\cot^{-1}\sqrt{2}\left(\cos\frac{\pi}{4}\right)\right] = \sin(\cot^{-1}1)$ $= \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$ **OR**	**I** **II**	1 1

(b)	As $(1,2) \in R$, but $(2,1) \notin R \Rightarrow R$ is not symmetric relation. $R_1 = \{(2,2), (2,1)\}$	**I** **II**	1 1
25.	If for two unit vectors $\vec{a}$ and $\vec{b}$, $\lvert \vec{a} + 2\vec{b} \rvert = \lvert 2\vec{a} - \vec{b} \rvert$, then find the angle between $\vec{a}$ and $\vec{b}$.		
Sol. 25	Given that $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert = 1$ $\lvert\vec{a} + 2\vec{b}\rvert = \lvert 2\vec{a} - \vec{b}\rvert \Rightarrow \lvert\vec{a} + 2\vec{b}\rvert^2 = \lvert 2\vec{a} - \vec{b}\rvert^2$ $\Rightarrow \lvert\vec{a}\rvert^2 + 4\lvert\vec{b}\rvert^2 + 4\vec{a}.\vec{b} = 4\lvert\vec{a}\rvert^2 + \lvert\vec{b}\rvert^2 - 4\vec{a}.\vec{b} \Rightarrow \vec{a}.\vec{b} = 0$ $\Rightarrow$ Angle between $\vec{a}$ and $\vec{b} = \frac{\pi}{2}$	**I** **II** **III** **IV**	½ ½ ½ ½

SECTION C

This section comprises 6 Short Answer (SA) type questions of 3 marks each.

26.	A survey was conducted on the patients who have undergone knee replacement surgeries.  It was found that, Robotic Knee replacement surgeries have 90% success rate. On a particular day, robotic surgery was performed on three patients, A, B and C, one after the other. Assuming that the success and failure of each surgery is independent of each other, find the probability that : (i) exactly one surgery is successful, (ii) at most two surgeries are successful.		
Sol. 26	$P(\text{success}) = \frac{9}{10}$ and $P(\text{not success}) = \frac{1}{10}$ (i) Probability that exactly one surgery is successful $= \frac{9}{10} \times \frac{1}{10} \times \frac{1}{10} \times 3 = \frac{27}{1000}$ (ii) Probability that all three surgeries are successful $= \frac{9}{10} \times \frac{9}{10} \times \frac{9}{10} = \frac{729}{1000}$ Probability that at most two surgeries are successful $= 1 - \frac{729}{1000} = \frac{271}{1000}$	**I** **II** **III** **IV**	½ 1 1 ½

27.	(a) Find : $$\int \frac{dx}{x^{1/2}+x^{1/3}}$$ **OR** (b) Find : $$\int \tan^{-1}\left(\frac{1-x}{1+x}\right) dx$$		
Sol. 27(a)	Put $x = t^6 \implies dx = 6t^5 dt$	**I**	1
	$\int \frac{dx}{x^{\frac{1}{2}}+x^{\frac{1}{3}}} = 6\int \frac{t^5}{t^3+t^2}dt = 6\int \frac{t^3}{t+1}dt$		
	$= 6\int \left(t^2 - t + 1 - \frac{1}{t+1}\right) dt$	**II**	1
	$= 6\left(\frac{t^3}{3} - \frac{t^2}{2} + t - \log\|t+1\|\right) + c$		
	$= 2x^{\frac{1}{2}} - 3x^{\frac{1}{3}} + 6x^{\frac{1}{6}} - 6\log\left\|x^{\frac{1}{6}}+1\right\| + c$	**III**	1
	OR		
(b)	$\int \tan^{-1}\left(\frac{1-x}{1+x}\right) dx$		
	Getting $\int \left(\frac{\pi}{4} - \tan^{-1}x\right) dx$	**I**	1
	$= \frac{\pi x}{4} - \left[x\tan^{-1}x - \int \frac{x}{1+x^2}dx\right]$	**II**	$1\frac{1}{2}$
	$= \frac{\pi x}{4} - x\tan^{-1}x + \frac{1}{2}\log(1+x^2) + c$	**III**	$\frac{1}{2}$
28.	Evaluate : $$\int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1+\sin 2x} dx$$		
Sol. 28	Let, $I = \int_0^{\frac{\pi}{2}} \frac{\sin^2 x}{1+\sin 2x}dx$ ………….(1)		
	Getting $I = \int_0^{\frac{\pi}{2}} \frac{\cos^2 x}{1+\sin 2x}dx$ ………….(2)	**I**	1
	On adding equations (1) and (2), we get		

	$2I = \int_0^{\frac{\pi}{2}} \frac{1}{1+\sin 2x}\,dx$	II	½
	$\Rightarrow \quad 2I = \int_0^{\frac{\pi}{2}} \frac{1-\sin 2x}{\cos^2 2x}\,dx = \int_0^{\pi/2} \sec^2 2x\,dx - \int_0^{\pi/2} \sec 2x \tan 2x\,dx$	III	½
	$\Rightarrow \quad 2I = \left[\frac{\tan 2x}{2} - \frac{\sec 2x}{2}\right]_0^{\pi/2} \quad \Rightarrow \quad I = \frac{1}{2}$	IV	1
29.	(a) Find : $\int \frac{\cos x}{(2+\sin x)(4+\sin x)}\,dx$ **OR** (b) Find : $\int \frac{x+3}{x^2+4x+5}\,dx$		
Sol. 29(a)	Put $\sin x = t \;\Rightarrow \cos x\,dx = dt$	I	1
	$\int \frac{\cos x}{(2+\sin x)(4+\sin x)}\,dx = \int \frac{1}{(2+t)(4+t)}\,dt$ $= \frac{1}{2}\int \left(\frac{1}{2+t} - \frac{1}{4+t}\right)dt = \frac{1}{2}\log\left\|\frac{2+t}{4+t}\right\| + c$	II	$1\frac{1}{2}$
	$= \frac{1}{2}\log\left\|\frac{2+\sin x}{4+\sin x}\right\| + c$	III	½
	OR		
(b)	$\int \frac{x+3}{x^2+4x+5}\,dx = \frac{1}{2}\int \frac{(2x+4)+2}{x^2+4x+5}\,dx$		
	$= \frac{1}{2}\int \frac{2x+4}{x^2+4x+5}\,dx + \int \frac{1}{x^2+4x+5}\,dx$	I	1
	$= \frac{1}{2}\log\|x^2+4x+5\| + \int \frac{1}{(x+2)^2+1}\,dx$	II	1
	$= \frac{1}{2}\log\|x^2+4x+5\| + \tan^{-1}(x+2) + c$	III	1
30.	(a) Find the general solution of the differential equation $(x^2 - y^2)\,dx + 2xy\,dy = 0$. **OR** (b) Solve the differential equation $\sin x \cos y\,dx + \cos x \sin y\,dy = 0$, given that $y = \frac{\pi}{4}$ when $x = 0$.		

Sol.30 (a)	$\frac{dy}{dx} = \frac{y^2 - x^2}{2xy}$ Put $y = vx$ and $\frac{dy}{dx} = v + x\frac{dv}{dx}$ Thus, $v + x\frac{dv}{dx} = \frac{v^2 - 1}{2v}$ $\Rightarrow x\frac{dv}{dx} = -\frac{1 + v^2}{2v}$ $\Rightarrow \int \frac{2v}{1+v^2}dv = -\int \frac{dx}{x}$ $\Rightarrow \log\|1 + v^2\| = -\log\|x\| + \log\ \|c\|$ $\Rightarrow 1 + v^2 = \frac{c}{x}$ $\Rightarrow x^2 + y^2 = cx$	**I** **II** **III** **IV**	1 ½ 1 ½
(b)	**OR** $\sin x \cos y\, dx + \cos x \sin y\, dy = 0$ $\Rightarrow \quad \frac{dy}{dx} = -\frac{\sin x \cos y}{\cos x \sin y}$ $\Rightarrow \int \tan y\, dy = -\int \tan x\, dx$ $\Rightarrow \log\|\sec y\| = -\log\|\sec x\| + \log\|c\|$ $\Rightarrow \sec x \sec y = c$ Put $x = 0, y = \frac{\pi}{4}$ we get $c = \sqrt{2}$ Hence particular solution is given by $\sec x \sec y = \sqrt{2}$	**I** **II** **III** **IV**	1 ½ 1 ½
31.	If $(\sin x)^y = y^{\cos x}$, then find $\frac{dy}{dx}$.		
Sol. 31	$(\sin x)^y = y^{\cos x} \Rightarrow y \log \sin x = \cos x \log y$ On diff. wrt x both sides, we get $\frac{y \cos x}{\sin x} + \log \sin x \frac{dy}{dx} = \frac{\cos x}{y}\frac{dy}{dx} - \log y . \sin x$ $\Rightarrow \frac{dy}{dx} = \frac{y(y \cos x + \log y . \sin^2 x)}{\sin x (\cos x - y \log \sin x)}$ or $\frac{(y \cot x + \sin x . \log y)y}{\cos x - y \log \sin x}$	**I** **II** **III**	1 $1\frac{1}{2}$ ½

SECTION D

This section comprises 4 Long Answer (LA) type questions of 5 marks each.

32.	(a) Represent the equations of lines l_1 and l_2 in vector form and check whether they are intersecting or not. $l_1 : \frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ $l_2 : \frac{x+1}{-1} = \frac{2-y}{-2} = \frac{z-5}{5}$ **OR** (b) Opposite sides of a square are along the lines : $\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$ $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$ Find the area of the square if direction ratios of other pair of opposite sides of the square are given by $\langle -3, 6, p \rangle$. Also, find the value of p.		
Sol. 32(a)	l_1: $\vec{r} = (-3\hat{i} + \hat{j} + 5\hat{k}) + \lambda(-3\hat{i} + \hat{j} + 5\hat{k})$	**I**	1
	l_2: $\vec{r} = (-\hat{i} + 2\hat{j} + 5\hat{k}) + \mu(-\hat{i} + 2\hat{j} + 5\hat{k})$	**II**	1
	$\overrightarrow{a_1} = -3\hat{i} + \hat{j} + 5\hat{k}$; $\overrightarrow{b_1} = -3\hat{i} + \hat{j} + 5\hat{k}$		
	$\overrightarrow{a_2} = -\hat{i} + 2\hat{j} + 5\hat{k}$; $\overrightarrow{b_2} = -\hat{i} + 2\hat{j} + 5\hat{k}$		
	$\overrightarrow{a_2} - \overrightarrow{a_1} = 2\hat{i} + \hat{j}$	**III**	1
	$\overrightarrow{b_1} \times \overrightarrow{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = -5\hat{i} + 10\hat{j} - 5\hat{k}$	**IV**	1
	$\because (\overrightarrow{a_2} - \overrightarrow{a_1}).(\overrightarrow{b_1} \times \overrightarrow{b_2}) = 0$	**V**	½
	$\therefore$ Both lines are intersecting.	**VI**	½
	OR		
(b)	$\overrightarrow{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$; $\overrightarrow{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$; $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$		
	$\overrightarrow{a_2} - \overrightarrow{a_1} = 2\hat{i} + \hat{j} - \hat{k}$	**I**	1
	$(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = 9\hat{i} - 14\hat{j} + 4\hat{k}$	**II**	1
	$\lvert(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b}\rvert = \sqrt{293}$	**III**	½
	Distance between given parallel lines $= \frac{\lvert(\overrightarrow{a_2} - \overrightarrow{a_1}) \times \vec{b}\rvert}{\lvert\vec{b}\rvert} = \frac{\sqrt{293}}{7}$	**IV**	½
	$\therefore$ Length of side of square $=$ distance between parallel lines		
	$= \frac{\sqrt{293}}{7}$ or $\frac{\sqrt{293}}{7}$ units		
	Area of square $= \frac{293}{49}$ or $\frac{293}{49}$ square units	**V**	1
	$\because$ Adjacent sides of square are perpendicular to each other.		
	$\therefore \quad -3 \times 2 + 6 \times 3 + 6p = 0 \quad \Rightarrow \quad p = -2$	**VI**	1

33. Show that a function $f : R_+ \to A \subset N$, defined as $f(x) = 4x^2 + 12x + 15$ is one-one. Find set A so that f is onto where $R_+ = [0, \infty)$. Also, find if there exists $a \in R_+$ such that $f(a) = 7$. Justify.

Sol. 33 f defined in the question is not a function.

So, full marks to be given for attempting the question. — **I** — 5

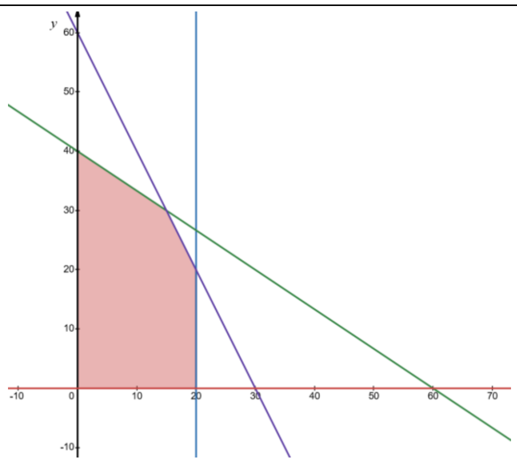
34. Solve the following Linear Programming Problem graphically :

Maximize $Z = 8x + 8y$

subject to the constraints

$$x - 20 \leq 0$$
$$2x + 3y \leq 120$$
$$2x + y \leq 60$$
$$x \geq 0,\ y \geq 0$$

Sol. 34



For correct graph and shaded region — **I** — 3

Corner Points	Value of Z=600x+400y
(20,0)	160
(20,20)	320
(15,30)	360
(0,40)	320
(0,0)	0

— **II** — $1\frac{1}{2}$

Hence, $Z_{max.} = 360$ at point (15,30). — **III** — ½

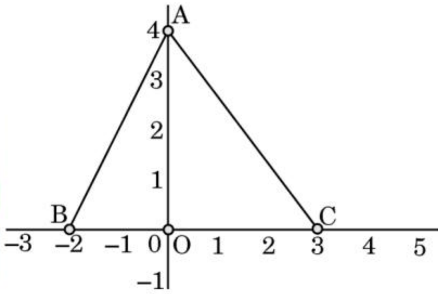
35.	(a) On the inauguration day of a new showroom, a lucky draw was organized and some vouchers of ₹ 1,000 and ₹ 500 were given to the lucky draw winners.		
			
	A total of 60 vouchers were given on the day. The number of ₹ 1,000 vouchers added to 3 times the number of ₹ 500 vouchers, gives 100. Express the given information as a system of linear equations in two variables. Hence, find the number of vouchers of each type by matrix method.		
	OR		
	(b) Given that $P = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$, $Q = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$ and $R = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$, find a matrix S such that PQ – RS is a null matrix.		
Sol. 35(a)	Let, number of ₹ 1000 vouchers $= x$		
	and number of ₹ 500 vouchers $= y$		
	Hence, $x + y = 60$ and $x + 3y = 100$	**I**	1
	In matrix form, $\begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 60 \\ 100 \end{bmatrix}$		
	$\Rightarrow$ AX = B where $A = \begin{bmatrix} 1 & 1 \\ 1 & 3 \end{bmatrix}, B = \begin{bmatrix} 60 \\ 100 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \end{bmatrix}$		
	$\Rightarrow \quad X = A^{-1}B \qquad \ldots\ldots\ldots (1)$	**II**	1
	$\lvert A \rvert = 2 \qquad \because \lvert A \rvert \neq 0 \Rightarrow$ A is an invertible matrix.	**III**	½
	adj $A = \begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}$ and $A^{-1} = \frac{1}{2}\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}$	**IV**	$1\frac{1}{2}$
	By eq. (1), $\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 3 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 60 \\ 100 \end{bmatrix} = \begin{bmatrix} 40 \\ 20 \end{bmatrix}$		
	On comparing, $x = 40$ and $y = 20$	**V**	1
	Hence, number of ₹ 1000 vouchers = 40		
	and number of ₹ 500 vouchers = 20		
	OR		
35(b)	Let $S = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$	**I**	½
	$PQ = \begin{bmatrix} 3 & 0 \\ 43 & 22 \end{bmatrix}$	**II**	1
	$RS = \begin{bmatrix} 2a + 5c & 2b + 5d \\ 3a + 8c & 3b + 8d \end{bmatrix}$	**III**	1

	Given that PQ − RS = Null matrix $\Rightarrow$ PQ = RS On comparing corresponding elements, $2a+5c=3, \quad 2b+5d=0, \quad 3a+8c=43, \quad 3b+8d=22$	IV	½
	By solving, $a=-191, \; b=-110, \; c=77, \; d=44$	V	$1\frac{1}{2}$
	Hence, $S=\begin{bmatrix}-191 & -110\\ 77 & 44\end{bmatrix}$	VI	½
	<u>Alternative Method:</u> Given that PQ − RS = null matrix $\Rightarrow$ RS = PQ	I	1
	$\begin{bmatrix}2 & 5\\ 3 & 8\end{bmatrix}S=\begin{bmatrix}3 & 0\\ 43 & 22\end{bmatrix}$	II	1
	$\Rightarrow S=\begin{bmatrix}2 & 5\\ 3 & 8\end{bmatrix}^{-1}\begin{bmatrix}3 & 0\\ 43 & 22\end{bmatrix}$	III	1
	$\Rightarrow S=\begin{bmatrix}8 & -5\\ -3 & 2\end{bmatrix}\begin{bmatrix}3 & 0\\ 43 & 22\end{bmatrix}$	IV	$1\frac{1}{2}$
	$\Rightarrow S=\begin{bmatrix}-191 & -110\\ 77 & 44\end{bmatrix}$	V	½

SECTION E

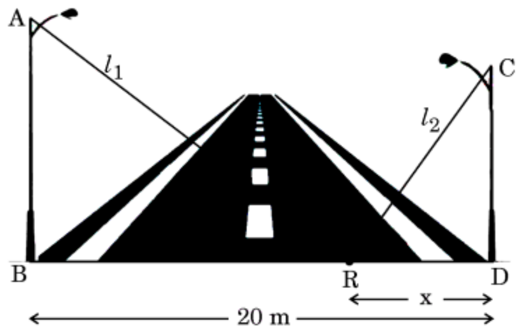
This section comprises 3 case-study based questions of 4 marks each.

	Case Study – 1		

<table>
<tr><td>36.</td><td>There is a triangular park in the society. The park is divided into two sections as shown in the figure.


In the region OAC, children are allowed to play games like cricket, football, while in the region AOB, activities which involve running are not allowed.
The vertices of the triangular park ABC are A(0, 4), B(– 2, 0) and C(3, 0).
Based on the above information, answer the following questions :
(i) Write the equation of the boundary line AB of the park.
(ii) Write the equation of the boundary line AC of the park.
(iii) (a) Using integration, find the area of region OAC, in which children are allowed to play cricket, football.
OR
(iii) (b) Using integration, find the area of region AOB.</td><td></td><td></td></tr>
<tr><td>Sol. 36(i)</td><td>Equation of the boundary line AB is $2x - y + 4 = 0$</td><td>I</td><td>1</td></tr>
<tr><td>(ii)</td><td>Equation of the boundary line AC is $4x + 3y = 12$</td><td>I</td><td>1</td></tr>
<tr><td>(iii)(a)</td><td>Area of region OAC $= \int_0^3 \frac{12-4x}{3}\,dx$
$= \frac{1}{3}[12x - 2x^2]_0^3 = 6$</td><td>I
II</td><td>1
1</td></tr>
<tr><td></td><td>OR</td><td></td><td></td></tr>
<tr><td>(iii)(b)</td><td>Area of region AOB $= \int_{-2}^{0} (2x+4)\,dx$
$= [x^2 + 4x]_{-2}^{0} = 4$</td><td>I
II</td><td>1
1</td></tr>
<tr><td></td><td>Case Study – 2</td><td></td><td></td></tr>
<tr><td>37.</td><td></td><td></td><td></td></tr>
</table>

Two vertical light poles of height 22 m and 16 m stand on the opposite sides of a 20 m wide road as shown below in the figure.



Two ladders of length l_1 and l_2 are placed from a common point R on the road at a distance of x m from the smaller pole.

Based on the above information, answer the following questions :

(i) Express $p(x) = l_1 + l_2$ in terms of x.

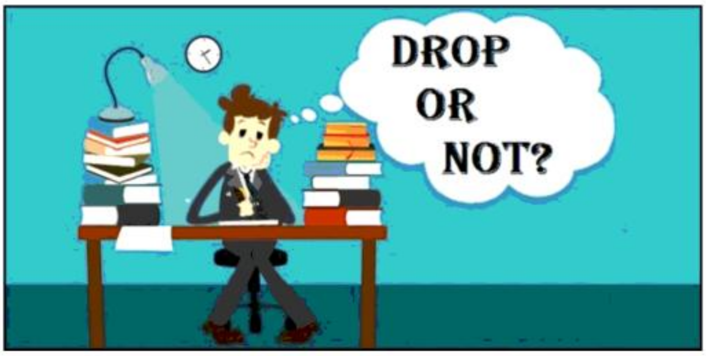
(ii) Find p′(x).

(iii) (a) Find the value of x for which $l_1^2 + l_2^2$ is minimum.

OR

(iii) (b) If the 22 m long pole is also replaced by a 16 m long pole, at what distance from either pole should the ladders be kept so that the sum of squares of lengths of ladders needed to reach the top of the pole is minimum ?

Sol. 37(i)	$p(x) = \sqrt{22^2 + (20-x)^2} + \sqrt{16^2 + x^2}$ **or** $p(x) = \sqrt{x^2 - 40x + 884} + \sqrt{x^2 + 256}$	**I**	1
(ii)	$p'(x) = \dfrac{x-20}{\sqrt{x^2 - 40x + 884}} + \dfrac{x}{\sqrt{x^2 + 256}}$	**I**	1
(iii)(a)	Let $q(x) = {l_1}^2 + {l_2}^2 = 2x^2 - 40x + 1140$	**I**	½
	$q'(x) = 4x - 40$	**II**	½
	Put $q'(x) = 0 \Rightarrow x = 10$	**III**	½
	$q''(x = 10) = 4 > 0 \Rightarrow q(x)$ is minimum at $x = 10$.	**IV**	½
	OR		
(iii)(b)	Let, ladder is kept at y m distance from one pole.		
	Suppose, $r(y) =$ sum of squares of lengths of ladders		
	$\Rightarrow r(y) = (16^2 + y^2) + [16^2 + (20-y)^2]$		
	$\Rightarrow r(y) = 2y^2 - 40y + 912$	**I**	½
	On diff. wrt y, we get $r'(y) = 4y - 40$	**II**	½

	Put $r'(y) = 0 \Rightarrow y = 10$ $r''(y) = 4 > 0 \Rightarrow r(y)$ is minimum at $y = 10$. Hence, the required distance of the ladder from each pole $= 10$ m	**III** **IV**	½ ½
	Case Study – 3		
38.	A survey was conducted to find out the success rate of students who qualified the entrance examination by dropping a year after class XII.  As per the data collected, 40% students appearing in the examination were dropouts and the remaining students were regular students of class XII. Of the dropouts, 5% qualify the examination while 10% of the regular students qualify the examination. Based on the above information, answer the following questions. (i) Find the probability that a student selected at random is a regular student. (ii) A student is selected at random from a group of dropout students. What is the probability that the student will not qualify the examination ? (iii) (a) A student selected at random qualified the examination. Find the probability that student is not a dropout. **OR** (iii) (b) A student selected at random did not qualify the examination. Find the probability that the student was a regular student.		
Sol.38(i)	Probability that a student selected at random is a regular student $= \frac{3}{5}$	**I**	1
38(ii)	Probability that the selected student from the group of dropout will not qualify the examination $= \frac{95}{100}$ or $\frac{19}{20}$	**I**	1
38(iii)(a)	Let A: The student has qualified the examination. E_1: The student appearing in the examination is dropout. E_2: The student is regular. Using Bayes' Theorem,		

	$$P\left(\frac{E_1'}{A}\right)=P\left(\frac{E_2}{A}\right)=\frac{\frac{60}{100}\times\frac{10}{100}}{\frac{40}{100}\times\frac{5}{100}+\frac{60}{100}\times\frac{10}{100}}$$ $$=\frac{3}{4}$$	**I** **II**	$1\frac{1}{2}$ ½
	OR		
38(iii)(b)	$$P\left(\frac{E_2}{A'}\right)=\frac{\frac{60}{100}\times\frac{90}{100}}{\frac{40}{100}\times\frac{95}{100}+\frac{60}{100}\times\frac{90}{100}}$$ $$=\frac{27}{46}$$	**I** **II**	$1\frac{1}{2}$ ½